Formalising Combinatorial Optimisation in Isabelle/HOL

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About Myself

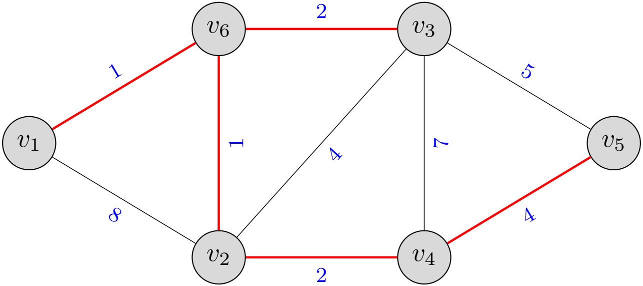
- 2nd year PhD student at King's College London

About My Work

- formalisation of combinatorial optimisation (CO) theory and algorithms
- formalisation = machine-readable/-processable/-checkable mathematics
- in the Isabelle/HOL prover
- CO: find an optimum solution for a problem that is based on a finite structure, e.g. graph

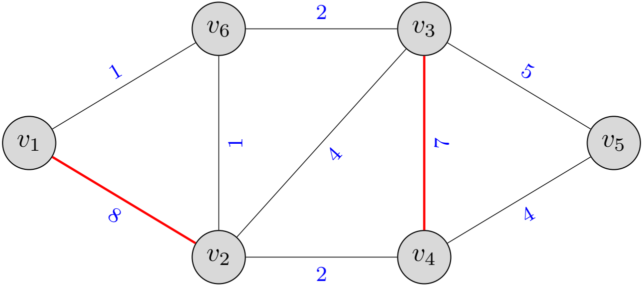
An Example: Minimum Spanning Tree

- undirected graph with edge costs
- spanning tree = acyclic and connected subgraph
- find a spanning tree that minimises accumulated costs



An Example: Matching

- given an undirected graph
- find a set of vertex-disjoint edges
- while aiming at an optimisation objective
- e.g. mere cardinality, or accumulated costs



Other Problems

- maximum flows
- minimum cost flows
- approximations for NP-hard problems

Why is formalising this interesting?

- classical optimisation problems with real-world applications:
  - shipping goods
  - kidney exchange
  - auctions and market design
  - Christofides' Heuristic to approximate TSP
- executable and verified code for CO algorithms
- optimality criteria: characterisation of optimum solutions by computationally easy properties
- techniques to facilitate or speed up computation: e.g. scaling, contraction of substructures or dynamic programming
- CO problems define structures in $\mathbb{R}^n$ with special properties
- more abstract perspective: matroids
- library together with collaborators (Abdulaziz, Rimpapa, Meenakshisundaram)

THANK YOU